

# Developing a stepwise approach to simulate a hammer mill through the Whiten model – the adherence for a gold ore

<http://dx.doi.org/10.1590/0370-44672018720186>

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## Abstract

Comminution represents a large portion of the capital and operating cost of a mineral processing plant. In 1983, Cohen estimated that comminution processes could account for 30% to 50% of the power consumption of the mill, and typically represents 50% of the operating costs of a mine. Therefore, its optimization is directly related to reduction of these operating costs. Among the crushing equipment, the hammer mill is one which is dedicated to operations that aim for a high reduction ratio with the controlled generation of fines. This crusher is recommended for friable and low abrasive ores presenting a high productive capacity. This study aims to develop a stepwise approach that allows the use of the classical crusher model (Whiten-Andersen) in modeling and simulation of circuits containing a hammer mill, simulating the resulting product according to variation of rotation speed within the equipment. The existing model for crushers developed by Whiten-Andersen considers the Perfect Mixing Model, which represents crushing through equations related to selection and breakage functions, that provide an equilibrium condition. This study aims to develop a stepwise approach that allows the use of the classical crusher model in modeling and simulation of circuits containing a hammer mill, by simulating the effect that the variation of rotation speed within the equipment implies in the generated product. The comparisons between the experimental and simulated data indicated that the model fits the data for both, P80 values and percentage passing in 19.1 mm sieve. The model created was validated based on specific experimental campaign.

**keywords:** crushing, hammer mill, modelling, simulation.

## 1. Introduction

Comminution represents a large portion of the capital and operating cost of a mineral processing plant. Cohen (1983) estimated that comminution processes could account for 30% to 50% of the power consumption of the mill, up to 70% for tough ores, and typically represents 50% of the operating costs of a mine. Therefore, its optimization is directly related to the reduction of these operating costs.

Napier-Munn (2006) defines crushing as the first mechanical stage for the comminution of a material in a mining operation, and the first stage inside a processing plant. In this process, the coarser particles are comminuted through the action of impact and compression forces (CHAVES, 2012). At this stage, the energy is applied almost individually to the particles resulting in a high energy per particle, although the total energy applied per unit mass is relatively low (KELLY & SPOTTISWOOD, 1982).

Amongst the crushing equipment, the hammer mill is one which is dedicated to operations that aim for a high reduction ratio

with the controlled generation of fines. This equipment is recommended for friable and low abrasive ores presenting high productive capacity (NAPIER-MUNN *et al.*, 2006).

In order to diagnose the operation of the plant and simulate changes in the operation, the use of phenomenological models in the mineral industry is highlighted. Such models consider the comminution equipment as an element of transformation of the particle size distribution of the feed. Amongst these models, the Population Balance Model (PBM) is the most popular (FOGGIATTO, 2009).

Developed in 1947 by Epstein the Population Balance Model (PBM) has been widely applied since its creation in both optimization and process control, such as in the design of facilities. The same model served as the basis for studies by Whiten (1972), who developed the perfect mixer model.

Delboni (2012) has described the development of a model for crushers in detail. In his text the author explains that the crushing operation model described here was proposed by Andersen (1988). This,

in turn, based his study on the concepts initially introduced by Whiten (1972). Like other phenomenological models of operation of comminution equipment, this incorporates two fundamental aspects: the characteristics of the material fragmentation and the peculiar characteristics of the process equipment.

The crushing process can be described as a sequence of selection phenomena followed by breakage events. Initially, every fragment is selected within the inner chamber of a crusher. The very fine fractions will be discharged directly, suffering no breakage. Larger fragments will be broken and the product will be classified, again being broken continuously until it is discharged through the lower opening of the equipment.

The Perfect Mixing Model represents the crushing process, through equations related to the selection and breaking functions, for the equilibrium condition.

The main equation of Whiten-Andersen model for crusher can be seen in Equation (1). (Whiten e White, 1979):

$$p_i = (I - S) \cdot (I - Q \cdot S)^{-1} \cdot f_i \quad (1)$$

where:  $I$  = unit matrix;  $S$  = classification function, diagonal matrix that contains the proportion of each granulometric fraction that will be comminuted;  $Q$  = appearance function, triangular matrix containing the granulometric distribution of each granulometric fraction after a breakage event;

The matrix representation of the model is very convenient, since the inflows and outflows of the equipment are expressed in vector form and the properties of the material and equipment defined according to the average sizes of each sieve interval.

Fragmentation in crushers is con-

$$t_{10} = A (1 - e^{-bE_{cs}}) \quad (2)$$

where:  $t_{10}$  = Percent passing through mesh equal to 10% of the original size of the fragment;  $E_{cs}$  = Specific energy applied to the ore fragment (kWh/t);  $A, b$  = Parameters dependent on the resistance to the fragmentation of the ore (determined by the DWT test).

This study aims to develop a stepwise approach that allows the use of the classical crusher model (Whiten-Andersen) in modeling and simulation of circuits containing

hammer mill, simulating the effect that the variation of rotation speed within the equipment implies in the generated product.

A similar study was conducted by several authors, but in different perspectives. Among them, we can highlight Nikolov (2002), who aims to simulate the product generated by an impact crusher through the variation in the rotation speed, calculating the power of the equipment and conse-

veniently represented by the parameter  $t_{10}$ , which is determined through the DWT test, which aims to determine the parameters of the parametric function between applied energy and the resulting fragmentation (Bergerman, 2009), according to equation (2) (Napier-Munn, 1996).

quential breakage, with a special function to predict the fines generation.

Another similar application for a different equipment can be seen in the study of Segura-Salazar *et al.* (2017), who applies a model based on the Whiten model for prediction of the product size in a VSI crusher and validates this model through Discrete Element Modeling analysis.

## 2. Materials and methods

The object of study was a crushing circuit belonging to the industrial gold ore processing plant of Córrego do Sítio I - HL (Heap Leaching) plant, located in Santa Bárbara - MG. The plant is part of the Córrego do

Sítio complex, operated by Anglo Gold Ashanti.

The development of the method used was based on industrial circuits and encompassed the following main components:

- Industrial plant sampling;
- Laboratory tests;
- Development of mathematical model of breakage;
- Scale-down e scale-up criteria;
- Simulation in computer simulator.

## 2.1 Sampling

Figure 1 shows the process flow diagram of the crushing circuit of Site I,

identifying and highlighting the sampling points selected for the campaign.

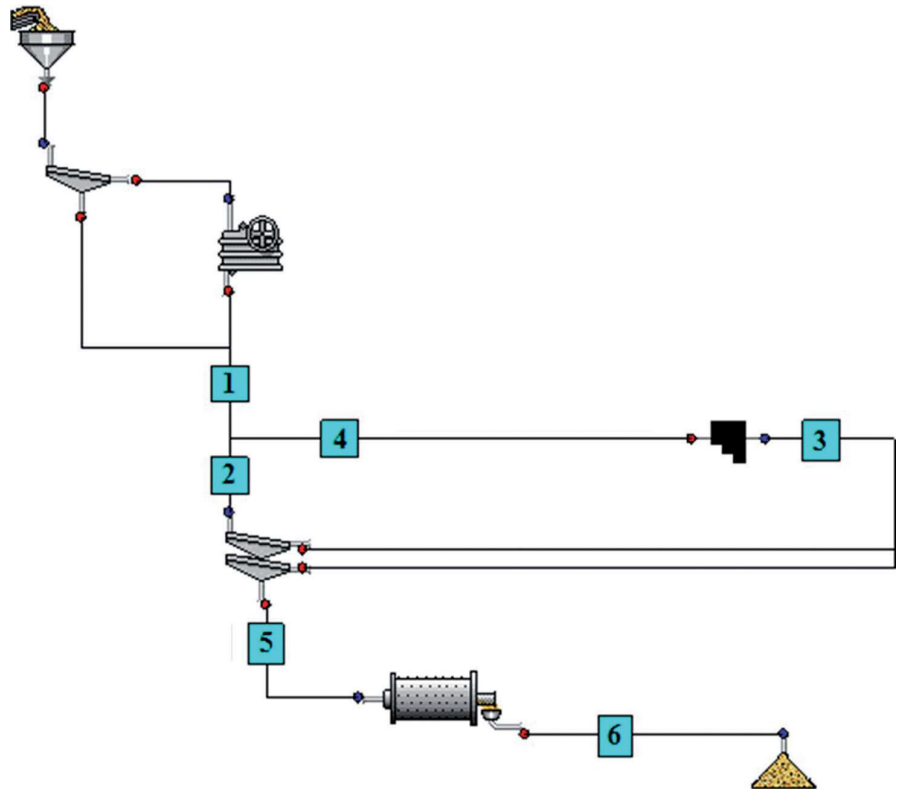


Figure 1  
Flow diagram of the CdSI  
circuit and selected sampling points.

The sampling points selected for the campaign were:

1. Primary crushing product;

2. Screening feed;

3. Combined screening oversize;

4. Hammer mill product;

5. Screening undersize;

6. Agglomerated product.

## 2.2. Technological characterization

After the sampling campaign, the samples, duly identified and conditioned, were sent to the LTM / EPUSP (Laboratory of Mineral Treatment of the Polytechnic School of the University of São Paulo) and to LSC / EPUSP (Laboratory of Simulation and Control of the Polytechnic School of the Univer-

sity of São Paulo) to be submitted to the following technological characterization tests:

- Particle size distribution (PSD) through sieving;
- Determination of the specific mass of solids through pycnometry tests;
- Determination of the bulk den-

sity of the ore;

- Determination of the Bond Work Index (WI);
- Determination of the Abrasion Index (AI);
- Drop Weight Tests (DWT) – determination of the Ecs vs  $t_{10}$  curve and Breakage Index.

## 2.3 Proposed model

After the sampling campaign, the sampled streams are sent to a technological characterization process in the laboratory and their consistency is verified. With this data and the notes from the sampling campaign, the mass balance is calculated.

The Base Case for mathematical modeling of the circuit is then generated with the mass balance and the data obtained in the technological characterization. The simulator used for mass balance closing, modeling and simulation was *JKSimMet* Version 6.0.1,

whose models were described in the present work.

The Appearance Function (3) parameters were set as follows:  $K1 = 0$ , with means that all particles can be broken,  $K2$  is the top size of the feed and  $K3 = 2.3$  as the classical approach from Whiten studies.

$$S(x) = 1 - \left( \frac{K2 - x}{K2 - K1} \right)^{K3} \quad (3)$$

It is assumed that a variation in the rotation speed of the hammer mill will im-

ply in a variation of the kinetic energy in the equipment and, consequently, a variation

in the specific breaking energy will occur, according to the following equation (4):

$$E_{CS} \sim E_{CIN} = M\omega^2 R^2 FP \quad (4)$$

where:  $E_{CS}$  = Specific breaking energy;  
 $E_{CIN}$  = Kinetic energy;  $M$  = Hammer mass (kg);  $\omega$  = Angular speed (Hz);  $R$  = Distance between axle and hammer (m);  
 FP = Power transfer factor - adopted

90%;  
 For the different rotations, without altering the other parameters of the hammer mill, a variation in the kinetic energy of impact is obtained by varying

$$E_{CS_{SIM}} = E_{CS_{CB}} \times \frac{(n_{SIM})^2}{(n_{CB})^2} \quad (5)$$

where:  $E_{CS_{SIM}}$  = Specific breaking energy of the simulation;  $E_{CS_{CB}}$  = Specific breaking energy of the Base Case;  $n_{SIM}$  = Simulated hammer rotation speed;  $n_{CB}$  = Rotation speed at the Base Case.

Thus, a new value is obtained for  $t_{10}$  associated to the new value of the hammers' rotation speed, according to Equation 4. The new fragmentation profile obtained for the new rotation is then applied to the

the rotation speed of the hammers, implying a variation in the specific breaking energy (5):

calibrated model and the simulation of the new value of the hammers' rotation speed is then performed, keeping as simulation basis the classic model of Whiten and Andersen, according to the flow chart of Figure 2.

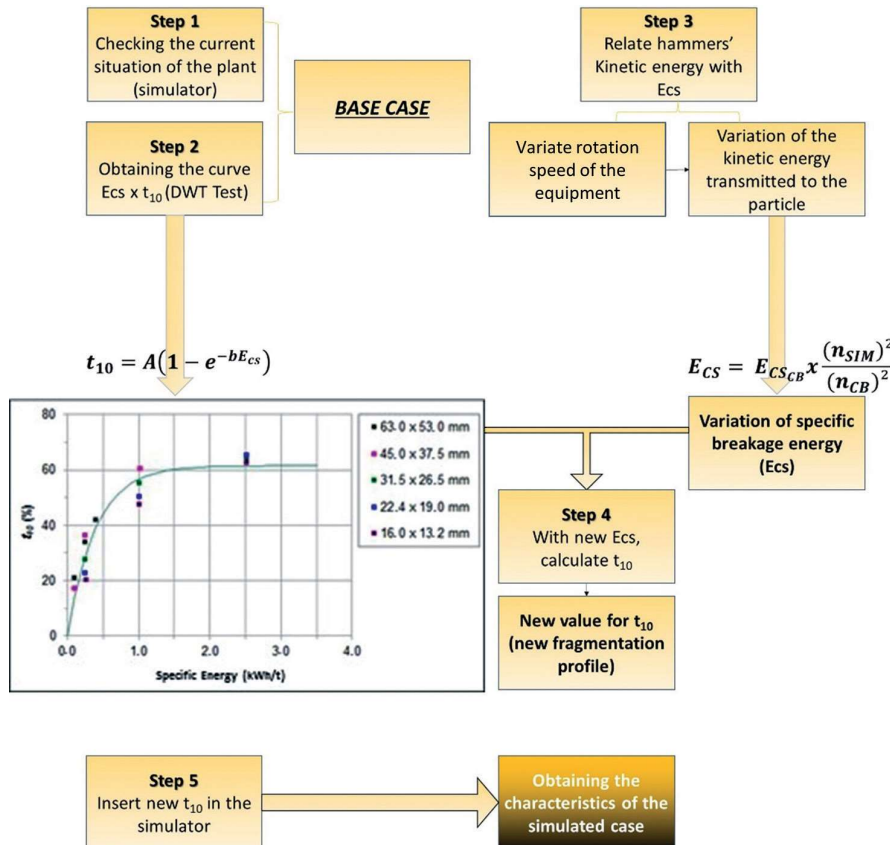


Figure 2  
Proposed model.

### 3. Results and discussion

In order to evaluate the adherence of the proposed model, bench tests with laboratory hammer mills were carried

out in the Laboratory of Ore Treatment of the Polytechnic School of the University of São Paulo (LTM-EPUSP) and

later simulation of the tested scenarios were performed. The obtained results were then compared.

#### 3.1. Technological characterization

A comprehensive technological characterization of the ore was conducted. The results can be seen in the Table 1.

Specific Mass	Bul Density (g/cm <sup>3</sup> )	WI (kWh/t)	AI	Breakage Index
2.67	1.40	9.3	0.092	156

Table 1  
Technological characterization of the ore.

Were the WI is the Grinding WI and the Breakage Index is defined as the product of the parameters A (61.5) and B (2.53) from DWT tests. Its value indicates

that the ore has extremely low resistance for crushing.

From the DWT tests, the fragmentation curve of the ore studied, ( $t_{10}$  vs

ECS) was obtained, as can be observed in Figure 3. The Base Case presented a  $t_{10}$  value of 5.851, with equipment operating at 900 rpm.

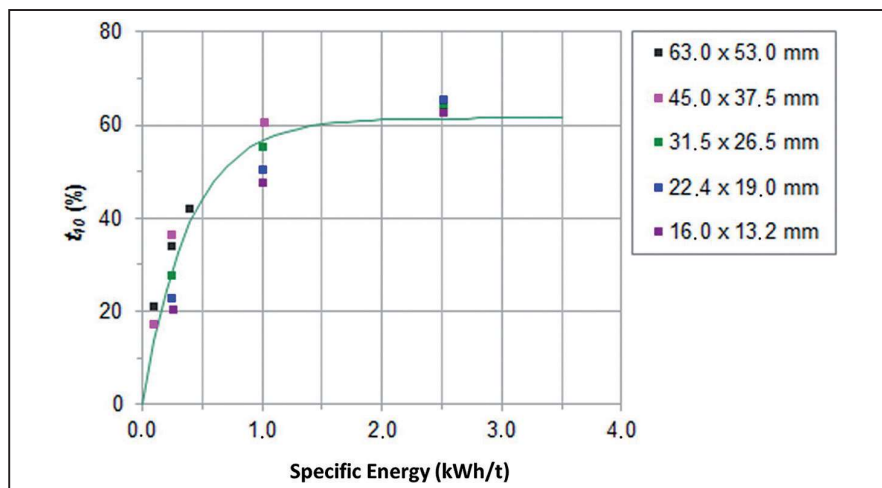


Figure 3  
 $t_{10}$  vs specific breakage energy curve.

The Breakage Function was also obtained from the DWT tests and its value can be observed in Table 2.

$t_{10}$	$t_{75}$	$t_{50}$	$t_{25}$	$t_4$	$t_2$
10	4.1	4.9	6.6	19.3	38.2
20	8.3	9.8	13.1	36.9	64.7
30	12.4	14.7	19.7	52.7	81.7

Table 2  
Breakage function.

The determination of the Particle Size Distribution (PSD) in the hammer

mill feed in the plant was conducted, and its result can be seen in Table 3.

Sieve (mm)	Accumulate % Passing
102	100
76.2	88.4
63.0	68.8
50.8	55.2
37.5	37.5
25.4	21.2
19.1	9.0
16.0	6.0
12.7	5.2
9.50	4.7
6.35	4.3
3.35	3.7
1.68	3.3
0.850	3.1
0.420	2.7
0.210	2.4

Table 3  
Feed PSD.

The characterization tests indicate that the ore that feeds the mill presents average toughness and extremely low

abrasiveness. The feed flow of the hammer mill, presents a large amount of fines. Such characteristics corroborate for the choice

of the hammer mill as the breakage equipment in the circuit.

### 3.2 Bench tests

The bench tests were performed in the equipment available in the LTM-USP, a hammer mill with 5 HP nominal power,

composed of 3 lines of hammers, each line composed of 1 hammer of 10 kg; its maximum rotation speed is 925 rpm. The equip-

ment allows variation of the rotation speed of the hammers for the various tests. Details of the equipment can be seen in Figure 4.





Figure 4  
LTM-USP Hammer mill.

Only the coarse amount of the ore (above 6.35 mm) were subjected to break-

age. The parameters analyzed in the bench tests were the product and feed particle

size distribution. The results can be seen in Table 4.

Scenario	Hammer rotation speed (rpm)	P <sub>80</sub> (mm)	Reduction Ratio	% Passing at 19.1 mm	% Passing at 6.35 mm
Feed	-	46.89	-	1.43	0
Scenario 1	279	43.93	1.07	6.84	1.66
Scenario 2	420	41.32	1.13	16.13	5.11
Scenario 3	560	40.07	1.17	26.69	8.76
Scenario 4	630	34.02	1.38	37.68	11.71
Scenario 5	701	34.65	1.35	35.02	11.44
Scenario 6	841	31.68	1.48	45.88	15.86
Scenario 7	926	27.10	1.73	58.59	22.29

Table 4  
Results from bench tests.

### 3.3 Simulation

After the bench tests, the modeling and simulation of the same tests were performed at the *JKSimMet* simulator, utilizing the hammer mill model developed in the Base Case, altering only the

$t_{10}$  parameter for the simulations.

According to the field measurements, the ratio between the hammer masses of the laboratory equipment and the industrial equipment was calculated

$$E_{cin} = M\omega^2 R^2$$

where: M = Hammer Mass (kg);  $\omega$  = Angular speed (Hz); R = Distance between axle and hammer (m).

Through the kinetic energy equation, and assuming that the kinetic energy is proportional to the specific breaking

at 0.5. The same ratio was obtained between the arm length of the laboratory and the industrial equipment. Being the calculation of the kinetic energy as given by (6):

(6)

energy (4), it follows that the energy and  $t_{10}$  ratios for the scenarios tested are those described in Table 5.

Scenario	Hammer rotation speed (rpm)	E <sub>cs</sub> Factor	E <sub>cs</sub>	t <sub>10</sub>
Base Case	900	0.900	0.0049	0.764
Scenario 1	279	0.186	0.0005	0.074
Scenario 2	420	0.296	0.0015	0.227
Scenario 3	560	0.449	0.0022	0.344
Scenario 4	630	0.542	0.0027	0.415
Scenario 5	701	0.646	0.0032	0.494
Scenario 6	841	0.886	0.0044	0.678
Scenario 7	926	1.052	0.0052	0.803

Table 5  
Simulation parameters.

A summary of the results of the simulations can be seen in Table 6.

Scenario	Hammer rotation speed (rpm)	P <sub>80</sub> (mm)	Reduction Ratio	% Passing at 19.1 mm	% Passing at 6.35 mm
Feed	-	46.89	-	1.43	0
Scenario 1	279	41.79	1.01	8.88	5.07
Scenario 2	420	39.32	1.08	15.57	8.47
Scenario 3	560	39.29	1.08	24.77	16.14
Scenario 4	630	34.60	1.23	30.78	17.91
Scenario 5	701	33.36	1.27	33.87	19.88
Scenario 6	841	31.28	1.36	42.18	25.66
Scenario 7	926	29.54	1.44	53.84	36.62

Table 6  
Simulation results.

### 3.3 Adherence between tests and simulations

In order to evaluate the adherence of the simulated results to the tests performed, the deviations of some

parameters were observed, comparing the value obtained in the test and the value obtained in the simulations. The

variance calculation method can be seen in equation (7).

$$S^2 = \frac{\sum_i^N (x_i - \bar{x})^2}{N - 1} \quad (7)$$

When comparing P<sub>80</sub> values, a variance (s<sup>2</sup>) of 0.72 and a standard deviation (s) of 0.85 was obtained between the tests. When comparing the values

of percentage passing at the 19.1 mm sieve, a variance of 3.89 and a standard deviation of 1.97 between the tests was obtained. Figure 5 shows the relation-

ship between those parameters and the hammers' rotation speed, for values tested and simulated.

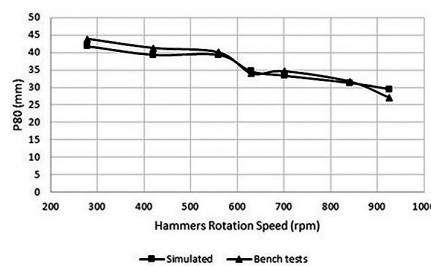
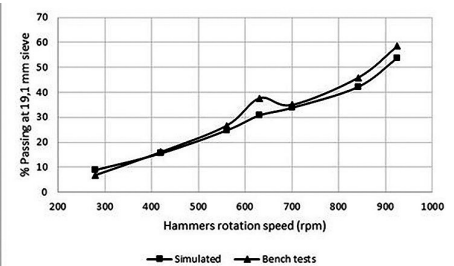


Figure 5  
P<sub>80</sub> and % passing at 19.1 mm sieve – bench tests versus simulated.



Analyzing the obtained data, it is observed that the model fits the data in the case studied as for the prediction of the product P<sub>80</sub>. As for the prediction of the passing material at 19.1 mm sieve, it is ob-

served that the model presents reasonable adherence, except for the value obtained in the simulation 4. This value is considered anomalous, since it is detached from the trend of the curve of experimental values

However, it is observed that, in general, the model overestimates the generation of fines (material passing in 6.35 mm sieve) by the hammer mill, as can be observed in Figure 6.

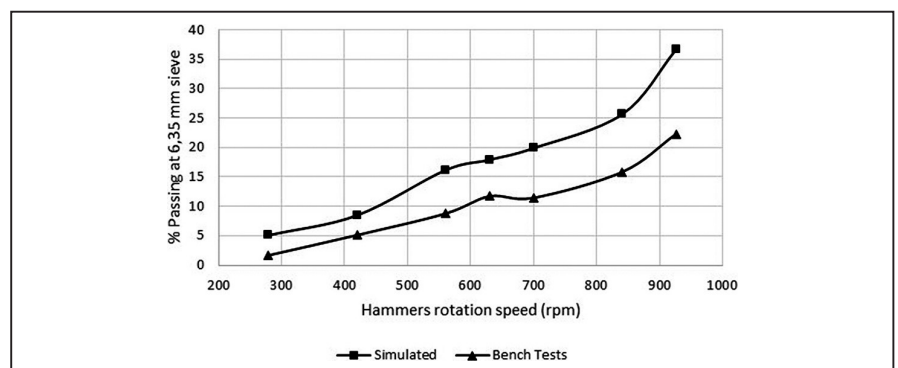


Figure 6  
Material passing at 6.35 mm sieve – bench tests versus simulated.

## 4. Conclusions

The comparisons between the experimental and simulated data indicated

adequate adhesion of the model for both, P<sub>80</sub> values and percentage passing

in 19.1 mm sieve. On the other hand, it was observed that the model overesti-

mates the generation of fines. The model created was, therefore, validated based on a specific experimental campaign.

When compared to the Nikolov (2002) model, the proposed model presents itself in a much simpler way of implementation with similar adhesion. The fines prediction of the presented

model can be supplemented by the Nikolov purpose.

The obtained results can also be complemented with Segura-Salazar et al. (2017) study. The proposed model acts in the breakage function while Segura-Salazar study acts in the selection function, for the same purpose,

with consistent results for both.

In order to increase the robustness of the created model, it is recommended to carry out a new sampling in the selected industrial circuit, after the implementation of the simulated changes, as well as to apply the model to other industrial plants.

## References

- ANDERSEN, J. S. *Development of a cone crusher model*. Austrália: University of Queensland, , 1988. 179 p. (M. Eng. Thesis).
- BERGERMAN, M. G. *Modelagem e simulação do circuito de moagem de sossego*. São Paulo: Escola Politécnica, Universidade de São Paulo, 2009. 207 p. (Mestrado em Ciências - Dissertação).
- CHAVES, A. P. *Britagem, peneiramento e moagem. Teoria e prática do tratamento de minérios*. (5. ed.). São Paulo: Ed. Signus, 2012. v. 3. p. 425 – 509.
- COHEN, H. E. Energy usage in mineral processing. *Transactions of the Institution of Mining and Metallurgy*. London, 1983. v. 92, p. 160-163. (Section C).
- FOGGIATTO, B. *Previsão de desempenho do circuito de moagem de carajás*. São Paulo: Escola Politécnica, Universidade de São Paulo, 2009. 230p. (Mestrado em Ciências – Dissertação).
- KELLY, E. G, SPOTTISWOOD, D. J. *Introduction to mineral processing*. New York: Ed. Wiley, 1982. 491 p.
- NAPIER-MUNN, T. J., MORREL S., MORRISON R.D., KOJOVIC T. *Mineral comminution circuits: their operation and optimization*. Queensland (JKMRC): 1996. 413 p.
- NIKOLOV, S. A performance model for impact crushers. *Minerals Engineering*, [s.l.], v. 15, n. 10, p.715-721, out. 2002. Elsevier BV. [http://dx.doi.org/10.1016/s0892-6875\(02\)00174-7](http://dx.doi.org/10.1016/s0892-6875(02)00174-7).
- SEGURA-SALAZAR, Juliana et al. Mathematical modeling of a vertical shaft impact crusher using the Whiten model. *Minerals Engineering*, [s.l.], v. 111, p.222-228, set. 2017. Elsevier BV. <http://dx.doi.org/10.1016/j.mineng.2017.06.022>.
- WHITEN, W. J. *Simulation and model building for mineral processing*. Australia: University of Queensland, 1972. p. 257 – 264. (Ph.d Thesis).
- WHITEN, W. J., WHITE, M.E., Modeling and simulation of high tonnage crushing plants. In: INTERNATIONAL MINERAL PROCESSING CONGRESS, 12. *Proceedings...* São Paulo, Brasil, 1979. p. 148–158.

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Received: 10 December 2018 - Accepted: 26 April 2019.



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